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A new method for identification of thermal boundary conditions in water-wall tubes of boiler furnaces

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ABSTRACT

In this paper, a method for determining fireside heat flux, heat transfer coefficient on the inner surface and temperature of water-steam mixture in water-wall tubes is developed. The unknown parameters are estimated based on the temperature measurements at a few internal locations from the solution of the inverse heat conduction problem. The non-linear least squares problem is solved numerically using the Levenberg–Marquardt method. The diameter of the measuring tube can be larger than the water-wall tube diameter. The view factor defining the distribution of the heat flux on the measuring tube circumference was determined using exact analytical formulas and numerically using ANSYS software. The method developed can also be used for an assessment of scale deposition on the inner surfaces of the water-wall tubes or slagging on the fire side.

1. Introduction

A proper understanding of combustion and heat transfer in furnaces and heat transfer processes on the water-steam side requires accurate measurement of heat flux which is absorbed by furnace walls [1–6]. There are three broad categories of heat flux measurements to the boiler water-walls: (1) portable heat flux meters inserted in inspection ports [7], (2) Gardon type heat flux meters welded to the sections of the boiler tubes [1,2,5,6], (3) tubular type instruments placed between two adjacent boiler tubes [2,4,8]. If a heat flux instrument is to measure the absorbed heat flux correctly, it must resemble the boiler tube as closely as possible so far as radiant heat exchange with the flame and surrounding surfaces is concerned. Two main factors in this respect are the emissivity and the temperature of the absorbing surface, but since the instrument will almost always be coated with ash, it is generally the properties of the ash and not the instrument that dominate the situation. Unfortunately, the thermal conductivity, can vary widely. Therefore, accurate measurements will only be performed if the deposit on the meter is representative of that on the surrounding tubes. The tubular type instruments known also as flux-tubes meet this requirement. In these devices the measured boiler tube wall temperatures are used for the evaluation of heat flux.

The measuring tube is fitted with two thermocouples in holes of known radial spacing r_1 and r_2. The thermocouples are led away to the junction box where they are connected differentially to give a flux related e.m.f [9,10].

The use of the one dimensional heat conduction equation for determining temperature distribution in the tube wall leads after rearrangements to the simple formula

$$\dot{q}_{\mathrm{m}} = \frac{k(f_1 - f_2)}{r_{\mathrm{o}} \ln(r_1/r_2)} \tag{1}$$

where f_1 and f_2 are measured wall temperatures at the locations r_1 and r_2, respectively. The accuracy of this equation is very low because of the circumferential heat conduction in the tube wall.

However, the measurement of the heat flux absorbed by water-walls with satisfactory accuracy is a challenging task. Considerable work has been done in recent years in this field [4,8,11,12].

Previous attempts to accurately measure the local heat flux to membrane water-walls in steam boilers failed due to calculation of inside heat transfer coefficients. The heat flux can be only determined accurately, if the inside heat transfer coefficient will be measured experimentally [4,8].

The other way to measure the local heat flux is based on the measurement of the water-wall temperature at two, three or at greater number of locations [4,8,11,12]. In the method presented in [4] the water steam temperature T_f was assumed to be equal to the measured metal temperature at the rear of the boiler tube. This assumption is allowed for new tubes without accumulated scale or corrosion deposits on the inner surface and if the tube wall thickness is small. The procedure presented in [8] involves the solution of the set of nonlinear algebraic equations. The number of unknown parameters to be determined is equal to the number

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Nomenclature

a	inside radius of boiler tube and flux-tube (m)
b	outside radius of flux-tube (m)
Bi	Biot number, $Bi = ha/k$
c	outside radius of boiler tube (m)
e	dimensionless tube pitch, $e = t/2c$
f_i	measured wall temperature at the i-th location (°C)
$\mathbf{f}$	vector of measured wall temperatures
h	heat transfer coefficient (W/(m^{2}K))
$\boldsymbol{I}_n$	identity matrix
$\boldsymbol{J}_m$	Jakobian matrix
k	thermal conductivity (W/(mK))
l	arbitrary length of boiler tube (m)
m	number of temperature measurement points
$\dot{q}_m$	heat flux to be determined (absorbed heat flux referred to the projected furnace wall surface) (W/m^2)
r	coordinate in cylindrical coordinate system or radius (m)
r_i	radial coordinate of the i-th thermocouple (m)
S	sum of measured and calculated temperature differences (K)
t	pitch of the wall tubes (m)
T	temperature (°C)
$\mathbf{T_m}$	vector of computed temperatures
u	ratio of the outside to the inside radius of the tube, $u = b/a$

Greek symbols

$\alpha, \beta, \gamma, \delta_1, \delta_2, \varepsilon$	angles shown in Fig. 1 (rad)
θ	temperature excess over the fluid temperature, $\theta = T - T_f$
φ	angular coordinate (rad)
φ_i	angular coordinate of the i-th thermocouple (rad)
ψ	view factor

Subscripts

in	inner
o	outer
i	number of temperature measurement point
f	fluid

of temperature measurement points. Thus, the method is very sensitive to small inaccuracies in wall temperature measurements. In the methods presented in [11,12] which are applied in heat flux measurements in fluidized bed boilers, the constant, independent of temperature thermal conductivity of the wall was assumed. The heat transfer coefficient between the tube inner surface and the coolant was calculated from the known correlations.

In this study, a numerical method for determining the heat flux in boiler furnaces, based on experimentally acquired interior tube temperatures, is presented. The tubular type instrument has been designed to provide a very accurate measurement of absorbed heat flux q_m, inside heat transfer coefficient h, and water steam temperature T_f. The number of temperature sensors (thermocouples) is greater than three because the additional information can aid in more accurate estimating the unknown parameters.

The temperature dependent thermal conductivity of the flux-tube material was accounted for.

The meter is constructed from a short length of boiler tube containing four thermocouples at the fire side part of the tube. The fifth thermocouple is located at the rear of the tube. The thickness of the flux-tube can be the same as the thickness of the boiler tube or can be larger.

In modern supercritical boilers [13] the tube wall thickness is up to 8 mm, so there is no need to increase the flux-tube thickness since the distance between thermocouples over the tube wall can be sufficient large to assure high accuracy of estimated parameters.

The presented method is appropriate for water-walls made of smooth tubes as well as for membrane water-walls (after removing the fins on the length of the flux tubes). The heat transfer conditions in adjacent boiler tubes have no influence on the temperature field in the flux-tube. The new method has advantages in terms of simplicity and flexibility.

2. Direct heat conduction problem

At first, the temperature distribution at the cross-section of the measuring tube will be determined, i.e. the direct problem will be solved.

Linear direct heat conduction problem can be solved using analytical method. The temperature distribution will be calculated numerically using the finite element method (FEM). In order to show accuracy of a numerical approach, the results obtained from numerical and analytical methods will be compared.

When material properties are assumed as temperature dependant the problem becomes non-linear and can be solved only numerically. The results obtained by finite element method will be presented.

2.1. Linear direct heat conduction problem

The following assumptions have been made:

- thermal conductivity of the measuring tube material is constant,
- heat transfer coefficient on the inner surface of the measuring tube and adjacent water-wall tubes does not vary on the tube circumference,
- rear side of the boiler setting is thermally insulated,
- diameter of the measuring tube can be larger than the diameter of the water-wall tubes,
- the outside surface of the measuring and water-wall tubes are irradiated by the plane flame surface, so the heat absorption on the tube fire side is non-uniform.

2.1.1. View factor determination

Radiation heat transfer between surfaces depends on the orientation of the surfaces relative to each other as well as their radiation properties and temperatures. To account for the effects of orientation on radiation heat transfer the view factor is calculated [14].

The view factor for interchange between an infinitesimal tube surface dA_i and a finite flame surface A_j follows directly from equation:

$$\psi = \frac{1}{2}(\sin\delta_1 + \sin\delta_2). \quad (2)$$

For angle interval $0 \leqslant \varphi \leqslant \varphi_1$ (Fig. 1) the following expression for ψ has been found:

$$\psi = \frac{1}{2}(1 + \cos\varphi), \quad 0 \leqslant \varphi \leqslant \varphi_1 \quad (3)$$

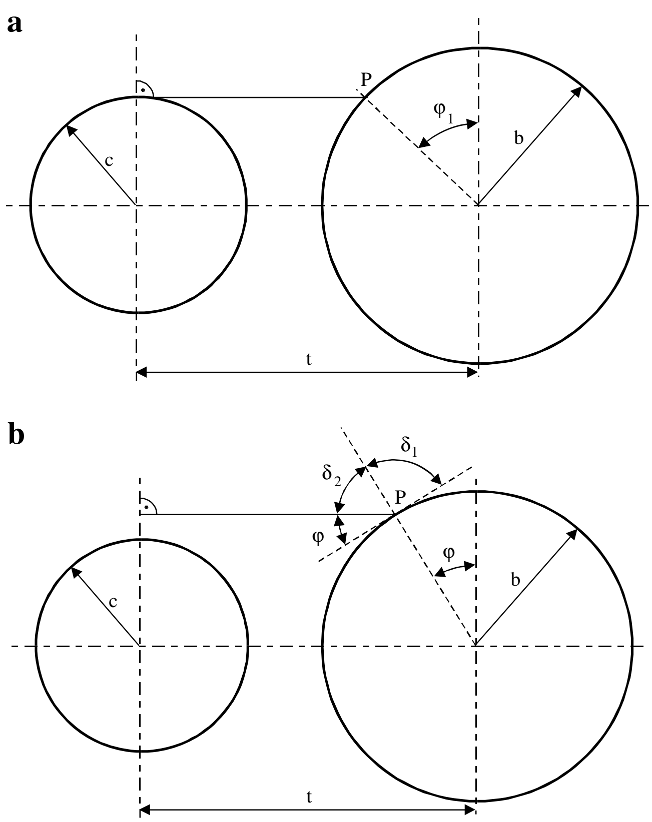

Fig. 1. View factor determination when the angle $0 \leqslant \varphi \leqslant \varphi_1$.

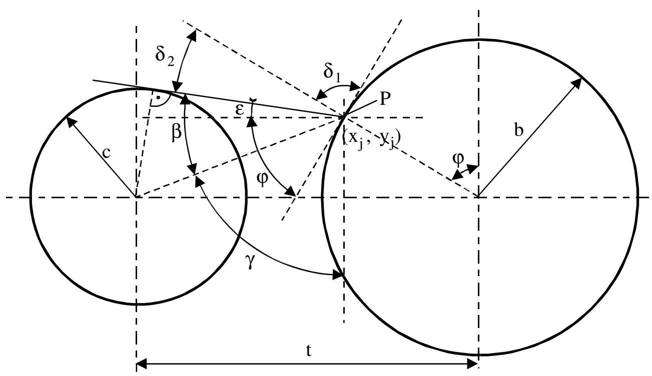

Fig. 2. View factor determination when the angle $\varphi_1 \leqslant \varphi \leqslant \pi/2$.

The view factor ψ for the angle interval $\varphi_1 \leqslant \varphi \leqslant \pi/2$ (Fig. 2) is given by Eq. (2) with

$$\delta_1 = \frac{\pi}{2}, \quad \delta_2 = \frac{\pi}{2} - (\varphi + \varepsilon), \quad \varepsilon = \beta + \gamma - \frac{\pi}{2},$$
$$\sin\beta = \frac{c}{\sqrt{(t-x_j)^2 + y_j^2}}, \quad \sin\gamma = \frac{t - x_j}{\sqrt{(t-x_j)^2 + y_j^2}}, \tag{4}$$
$$\frac{x_j}{b} = \cos\left(\frac{\pi}{2} - \varphi\right), \quad \frac{y_j}{b} = \sin\left(\frac{\pi}{2} - \varphi\right).$$

The symbols used in Eqs. (2) and (4) are shown in Fig. 2.

Eqs. (2) and (4) are valid for angle $\varphi_1 \leqslant \varphi \leqslant \pi/2$. The measuring tube surface is irradiated over the angle interval $0 \leqslant \varphi \leqslant \varphi_2$ while the rear part of the tube $\varphi_2 \leqslant \varphi \leqslant \pi$ does not receive any radiation from the fire side (Fig. 3).

The limiting angle φ_2(Fig. 3) can be determined, as follows:

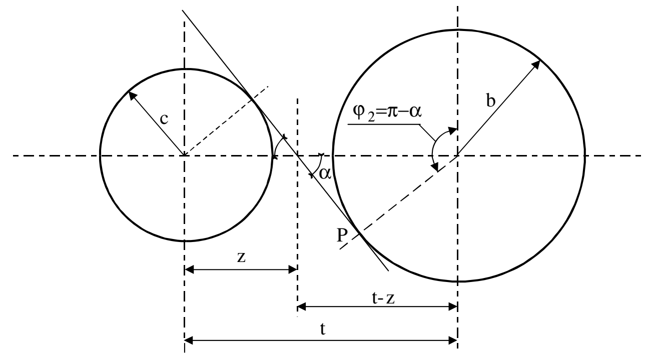

Fig. 3. Calculation of limiting angle φ_2.

$$\sin\alpha = \frac{c}{z} = \frac{b}{t - z}. \tag{5}$$

Solving Eq. (5) gives

$$z = \frac{ct}{b + c}. \tag{6}$$

The limiting angle φ_2 is given by a simple expression

$$\varphi_2 = \pi - \alpha = \pi - \arcsin\frac{c}{z} = \pi - \arcsin\frac{b+c}{t} \tag{7}$$

When $\pi/2 \leqslant \varphi \leqslant \varphi_2$ (Fig. 4) Eqs. (2) and (4) have to be changed to the form

$$\psi = \frac{1}{2}(\sin\delta_1 - \sin\delta_2) \tag{8}$$

where

$$\delta_1 = \frac{\pi}{2}, \quad \delta_2 = \varepsilon + \varphi - \frac{\pi}{2}, \quad \varepsilon = \beta + \gamma - \frac{\pi}{2}$$
$$\sin\beta = \frac{c}{\sqrt{(t-x_j)^2 + y_j^2}}, \quad \sin(\pi - \gamma) = \frac{t - x_j}{\sqrt{(t-x_j)^2 + y_j^2}} \tag{9}$$
$$\gamma = \pi - \arcsin\left(\frac{t - x_j}{\sqrt{(t-x_j)^2 + y_j^2}}\right)$$
$$x_j = b\cos\left(\varphi - \frac{\pi}{2}\right) = b\cos\left(\frac{\pi}{2} - \varphi\right),$$
$$y_j = -b\sin\left(\varphi - \frac{\pi}{2}\right) = b\sin\left(\frac{\pi}{2} - \varphi\right)$$

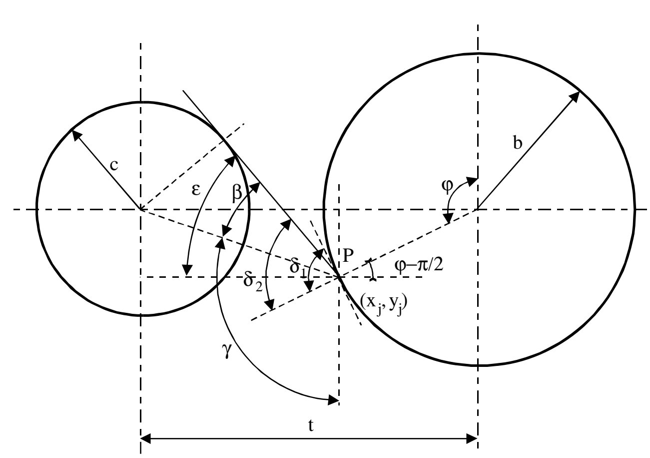

Fig. 4. View factor determination when the angle φ is larger than $\pi/2$ and smaller than the limiting angle φ_2.

Radiation leaving the flame surface reaches also the boiler setting. Heat flow rate from the flame to the boiler setting is given by:

$$\dot{Q}_{bs} = \dot{q}_m t l - 2\int_0^{\varphi_2} \dot{q}_m \psi(\varphi) c \, d\varphi \; l \tag{10}$$

The mean value of the heat flux to the boiler setting may be expressed as:

$$\dot{q}_{bs} = \frac{\dot{Q}_{bs}}{tl} = \dot{q}_m - \frac{\dot{q}_m}{e}\int_0^{\varphi_2} \psi(\varphi) d\varphi = \dot{q}_m(1-\psi_{fw}) = \dot{q}_m \psi_{bs} \tag{11}$$

where

$$e = \frac{t}{2c}, \quad \psi_{fw} = \frac{1}{e}\int_0^{\varphi_2} \psi(\varphi) d\varphi$$

Heat flux $\dot{q}_{bs}$ is reflected towards the rear side of the water-wall tube and flame.

When $c = b$ then ψ_{bs} reduces to:

$$\psi_{bs} = \frac{\sqrt{e^2-1} - arctg\sqrt{e^2-1}}{e} \tag{12}$$

The view factor for the radiation heat exchange between boiler setting and rear side of the measuring tube can be calculated in similar way as for the forward part.

2.1.2. Analytical determination of temperature distribution

Tube temperature distribution is expressed by heat conduction equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial\theta}{\partial r}\right) + \frac{1}{r}\frac{\partial}{\partial\varphi}\left(\frac{k}{r}\frac{\partial\theta}{\partial\varphi}\right) = 0 \tag{13}$$

and boundary conditions

$$k\frac{\partial T}{\partial r}\bigg|_{r=b} = \dot{q}_m\psi(\varphi) \tag{14}$$

$$k\frac{\partial\theta}{\partial r}\bigg|_{r=r_{in}} = h\theta|_{r=a} \tag{15}$$

where θ is the temperature excess of the tube T above the temperature of the medium T_f, i.e. $\theta = T - T_f$. Heat flux over the tube circumference can be approximated by the Fourier polynomial

$$\dot{q}_m\psi(\varphi) = \dot{q}_0 + \sum_{n=1}^{\infty} \dot{q}_n \cos(n\varphi) \tag{16}$$

where

$$\dot{q}_0 = \frac{1}{\pi}\int_0^{\pi} \dot{q}_m\psi(\varphi)d\varphi, \quad \dot{q}_n = \frac{2}{\pi}\int_0^{\pi} \dot{q}_m\psi(\varphi)\cos(n\varphi)d\varphi, \quad n = 1, \ldots \tag{17}$$

In conformity with the separation of variables method, the solution is searched for in the form

$$\theta(r,\varphi) = U(r) \times V(\varphi). \tag{18}$$

Substituting Eq. (18) into Eq. (13) gives

$$r^2U''V + rU'V + UV'' = 0. \tag{19}$$

After division of Eq. (19) by UV and separation of variables, one obtains

$$\frac{r^2U'' + rU'}{U} = -\frac{V''}{V}. \tag{20}$$

Since r and φ are independent variables, equality (20) occurs only when its both sides are equal to the same constant. If the constant were negative, the solution $V(\varphi)$ would then contain exponential functions and the periodic boundary condition (14) could not be satisfied. Separation constant, therefore, must be either a positive integral number or zero. If one assumes that both sides of Eq. (20) are equal to n^2, one obtains

$$r^2U'' + rU' - n^2U = 0, \tag{21}$$

$$V'' + n^2V = 0, \quad n = 0, 1, \ldots \tag{22}$$

In the case of a circular-symmetrical load only $\dot{q}_0 \neq 0$, whereas $\dot{q}_1 = \dot{q}_2 = \ldots = 0$. For $n = 0$, the solution of Eqs. (21) and (22) has the form

$$U(r) = A_0' + B_0' \ln r \tag{23}$$

and

$$V(\varphi) = C_0' + D_0'\varphi. \tag{24}$$

Due to the axis-symmetrical load $D_0' = 0$, and the product $U(r)V(\varphi)$ can be written in the form

$$U(r)V(\varphi) = A_0 + B_0 \ln r \quad \text{for } n = 0, \tag{25}$$

where, $A_0 = A_0'C'$, $B_0 = B_0'C_0'$.

For $n \geqslant 1$, the solution of Eqs. (21) and (22) has the forms

$$U(r) = A_n'r^n + B_n'r^{-n}, \tag{26}$$

$$V(\varphi) = C_n' \cos n\varphi + D_n' \sin n\varphi. \tag{27}$$

Due to the symmetrical heating of the water-wall tube assumed in the boundary condition (16) we have $D_n' = 0$ and product $U(r)V(\varphi)$ can be written as follows

$$U(r)V(\varphi) = (C_nr^n + D_nr^{-n})\cos\, n\varphi, \quad n \geqslant 1, \tag{28}$$

where, $C_n = A_n'C_n'$ and $D_n = B_n'C_n'$.

Then, expression (18), which describes the distribution of excess temperature in the tube, has the form

$$\theta(r,\varphi) = A_0 + B_0 \ln r + \sum_{n=1}^{\infty}(C_nr^n + D_nr^{-n})\cos n\varphi. \tag{29}$$

After substituting Eq. (29) into boundary conditions (14) and (15), one can determine constants, which can be written after transformation in the following form:

$$A_0 = \frac{\dot{q}_0 b}{\lambda}\left(\frac{1}{Bi} - \ln a\right), \tag{30}$$

$$B_0 = \frac{\dot{q}_0 b}{\lambda}, \tag{31}$$

$$C_n = \frac{\dot{q}_n b}{k}\,\frac{\frac{1}{n}u^n(Bi+n)\frac{1}{a^n}}{Bi(u^{2n}+1) + n(u^{2n}-1)}, \tag{32}$$

$$D_n = -\frac{\dot{q}_n b}{k}\,\frac{\frac{1}{n}u^n(Bi-n)a^n}{Bi(u^{2n}+1) + n(u^{2n}-1)}, \tag{33}$$

where, $u = b/a$, $Bi = ha/k$.

Eq. (29) can be used for the temperature calculation when all the boundary conditions are known.

3. Inverse problem

In the inverse heat conduction problem three parameters are to be determined:

- absorbed heat flux referred to the projected furnace wall surface: $x_1 = q_m$,
- heat transfer coefficient on the inner surface of the boiler tube: $x_2 = h$,
- fluid bulk temperature: $x_3 = T_f$.

These parameters appear in boundary conditions (14) and (15) and will be determined based on the wall temperature measurements at m internal points (r_i, φ_i)

$$T(r_i, \varphi_i) = f_i, \quad i = 1, \ldots, m, \quad m \geqslant 3. \tag{34}$$

In a general case, the unknown parameters: $x_1, \ldots, x_n$ are determined by minimizing sum of squares

$$S = (\mathbf{f} - \mathbf{T_m})^{\mathrm{T}}(\mathbf{f} - \mathbf{T_m}), \tag{35}$$

where $\mathbf{f} = (f_1, \ldots, f_m)^{\mathrm{T}}$ is the vector of measured temperatures, and $\mathbf{T_m} = (T_1, \ldots, T_m)^{\mathrm{T}}$ the vector of computed temperatures $T_i = T(r_i, \varphi_i)$, $i = 1, \ldots, m$.

The parameters $x_1, \ldots, x_n$, for which the sum (35) is minimum are determined using the Levenberg–Marquardt method [15]. The parameters, $\boldsymbol{x}$, are calculated by the following iteration:

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \boldsymbol{\delta}^{(k)}, \quad k = 0, 1, \ldots \tag{36}$$

where

$$\boldsymbol{\delta}^{(k)} = \left[(\mathbf{J}_m^{(k)})^T \mathbf{J}_m^{(k)} + \mu^{(k)} \mathbf{I}_n\right]^{-1} (\mathbf{J}_m^{(k)})^T \left[\mathbf{f} - \mathbf{T}_m(\mathbf{x}^{(k)})\right]. \tag{37}$$

The Jacobian $\mathbf{J}_m$ is given by

$$\mathbf{J}_m = \frac{\partial \mathbf{T}_m(\mathbf{x})}{\partial \mathbf{x}^T} = \left[\left(\frac{\partial T_i(\mathbf{x})}{\partial x_j}\right)\right]_{m \times n} \quad i = 1, \ldots, m \quad j = 1, \ldots, n. \tag{38}$$

The symbol $\mathbf{I}_n$ denotes the identity matrix of $n \times n$ dimension, and $\mu^{(k)}$ the weight coefficient, which changes in accordance with the algorithm suggested by Levenberg and Marquardt. The upper index T denotes the transposed matrix. Temperature distribution $T(r, \varphi, \mathbf{x}^{(k)})$ is computed at each iteration step using Eq. (29).

After a few iteration we obtain a convergent solution.

4. The uncertainty of the results

The uncertainties of the determined parameters $\mathbf{x}^*$ will be estimated using the error propagation rule of Gauss [15,16]. The propagation of uncertainty in the independent variables: measured wall temperatures f_j, $j = 1, \ldots m$, thermal conductivity k, water-steam temperature T_∞, radial and angular positions of temperature sensors r_j, φ_j, $j = 1, \ldots m$ is estimated from the following equation

$$2\sigma_{x_i} = \left[\sum_{j=1}^{m}\left(\frac{\partial x_i}{\partial f_j} 2\sigma_{f_j}\right)^2 + \left(\frac{\partial x_i}{\partial k} 2\sigma_k\right)^2 + \left(\frac{\partial x_i}{\partial T_\infty} 2\sigma_{T_\infty}\right)^2 + \sum_{j=1}^{m}\left(\frac{\partial x_i}{\partial r_j} 2\sigma_{r_j}\right)^2 + \sum_{j=1}^{m}\left(\frac{\partial x_i}{\partial \varphi_j} 2\sigma_{\varphi_j}\right)^2\right]^{1/2} \quad i = 1, 2, 3 \tag{39}$$

The 95% uncertainty in the estimated parameters can be expressed in the form

$$x_i = x_i^* \pm 2\sigma_{x_i}, \tag{40}$$

where $x_i^*, i = 1, 2, 3$ represent the value of the parameters obtained using the least squares method.

The sensitivity coefficients $\partial x_i/\partial f_j$, $\partial x_i/\partial k$, $\partial x_i/\partial T_\infty$, $\partial x_i/\partial r_j$, and $\partial x_i/\partial \varphi_j$ in the expression (39) were calculated by means of the numerical approximation using central difference quotients.

5. Examples of identification of thermal boundary conditions in water-wall tubes

Two examples will be presented. Firstly, a linear inverse heat conduction problem will be solved, assuming that material properties are temperature independent. Based on this assumption the temperature data necessary for inverse solution can be obtained from analytical formula. Next, a non-linear inverse heat conduction problem will be solved based on temperatures generated in a numerical way.

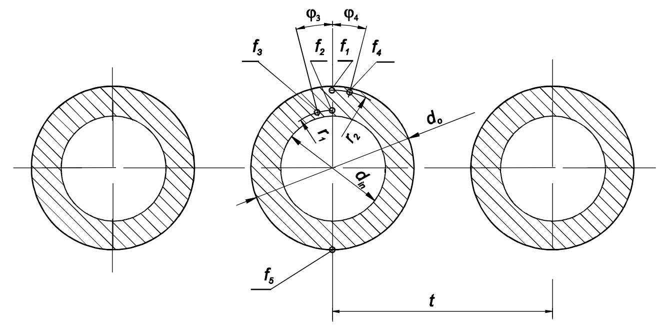

Fig. 5. An example of locations of temperature measurement points at water-wall tube ($\varphi_3 = \varphi_4 = 15°$, $r_1 = 26$, $r_2 = 29$ mm).

5.1. Linear inverse heat conduction problem

Consider a water-wall tube with the following parameters:

- outside radius $r_o = 30$ mm,
- inside radius $r_{in} = 25$ mm,
- pitch of the water-wall tubes $t = 80$ mm,
- thermal conductivity $k = 40.5$ W/(mK).

The water-wall tube temperature is measured by thermocouples located at five points presented in Fig. 5.

Using equations derived in chapter 2.1.1, the view factor distribution on the outer surface of water-wall tube can be calculated analytically. Additionally, the view factor distribution was determined using the finite element method (FEM) using ANSYS code [17]. Comparison of analytical and numerical results is presented in Fig. 6.

Next, the measured five temperatures were artificially generated using the analytical method presented in chapter 2.1.2.

The following parameters were considered:

- $\dot{q}_m = 220135.3$ W/m^2,

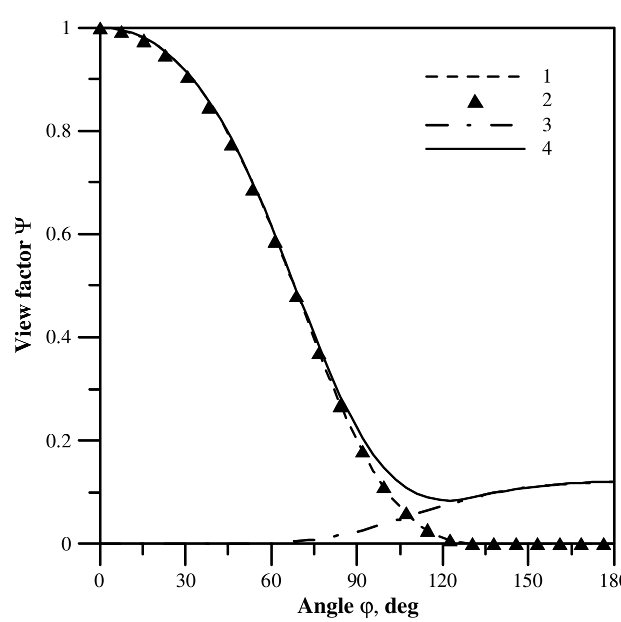

Fig. 6. View factor distribution on the outer surface of water-wall tube: (1) exact view factor for furnace radiation, (2) view factor for furnace radiation calculated by FEM, (3) view factor from boiler setting, (4) total view factor accounting radiation from furnace and boiler setting.

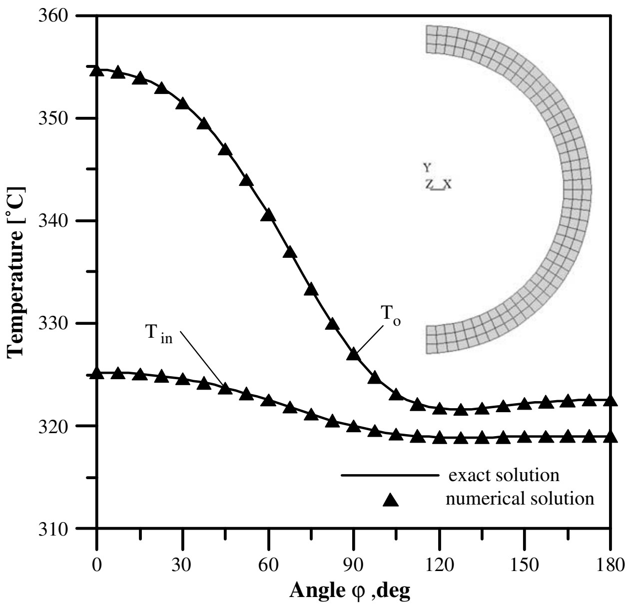

Fig. 7. Temperature distribution on the inner and outer surface of the water-wall tube calculated by exact and numerical methods.

- $h = 37105.5 \mathrm{W}/(m^2\mathrm{K})$,
- $T_f = 318.2$ °C.

When the heat flux on the outer surface in assumed as:

$$\dot{q}(\varphi) = \dot{q}_0 + \sum_{i=1}^{n} \dot{q}_i \cos(i\varphi)$$

coefficients can be determined using method of expansion into a Fourier series. For $n = 6$ the following coefficients were obtained:

$$\dot{q}_0 = 92093.6 \text{ W/m}^2, \dot{q}_1 = 103052.0 \text{ W/m}^2, \quad \dot{q}_2 = 36572.4 \text{ W/m}^2, \quad \dot{q}_3 = -6533.4 \text{ W/m}^2,$$
$$\dot{q}_4 = -5795.7 \text{ W/m}^2, \quad \dot{q}_5 = 471.8 \text{ W/m}^2,$$
$$\dot{q}_6 = 347.2 \text{ W/m}^2.$$

Temperature distribution on the inner and outer surface of the water-wall tube calculated by exact and numerical methods is presented in Fig. 7 and in Table 1. The relative error E between exact and numerical temperature values was calculated as

$$E = \frac{(T_{ex} - T_{num})}{T_{ex}} 100\%$$

where T_{ex} stands for the exact value and T_{num} for numerical value of temperature

Numerical calculations were carried out by the FEM. The division of domain into finite elements is shown in Fig. 7.

Artificially generated temperatures, necessary for identification of thermal boundary conditions in water-wall tubes, are as follows:

$$f_1 = 349.17 \text{ °C}, \quad f_2 = 331.52 \text{ °C}, \quad f_3 = 331.24 \text{ °C},$$
$$f_4 = 348.52 \text{ °C}, \quad f_5 = 322.55 \text{ °C}.$$

The inverse problem is solved iteratively, starting form initial values:

$$\dot{q}_m^{(0)} = 150,000 \text{ W/m}^2, \; h^{(0)} = 20,000 \text{ W/(m}^2\text{K)}, \; T_f^{(0)} = 300 \text{ °C}.$$

After 13 iterations the following values were founded:

$$x_1^* = \dot{q}_m^* = 220135.38 \text{ W/m}^2, \quad x_2^* = h^*$$
$$= 37107.54 \text{ W/(m}^2\text{K)}, \quad x_3^* = T_f^* = 318.2 \text{ °C}.$$

When compared with exact values:$\dot{q}_m = 220135.3$ W/m^2, $h = 317105.5$ W/(m^{2}K), $T_f = 318.2$ °C the accuracy of presented method can be estimated.

In order to show the influence of the measurement errors on the determined thermal boundary parameters, the 95% confidence intervals were calculated. The following uncertainties of the measured values were assumed (at a 95% confidence interval):

$$2\sigma_{f_j} = \pm 0.2 \text{ K}, \quad j = 1, \ldots, 5, \quad 2\sigma_k = \pm 0.5 \text{ W/(mK)}, \quad 2\sigma_{r_j}$$
$$= \pm 0.1 \text{ mm}, \quad 2\sigma_{\varphi_j} = \pm 1\text{rd}, \quad j = 1, \ldots, 5.$$

The uncertainties (95% confidence interval) of the coefficients x_i were determined using the error propagation rule formulated by Gauss.

The calculation yielded the following results: $x_1 = 220135 \pm 8138$ W/m^2, $x_2 = 37108 \pm 4536$ W/(m^{2}K), $x_3 = 318.2 \pm 0.25$ °C.

The accuracy of the results obtained is very satisfactory.

5.2. Non-linear direct heat conduction problem

Consider a water-wall tube with the following parameters:

- outside radius $r_o = 30$ mm,
- inside radius $r_{in} = 25$ mm,
- pitch of the wall tubes $t = 80$ mm.

Thermal conductivity of the tube material is temperature dependent:

$$k(T) = 48.51 + 8.078 \times 10^{-4} \times T - 6.296 \times 10^{-5} \times T^2$$
$$+ 4.016 \times 10^{-8} \times T^3, \text{ W/(mK)}$$

where T is given in °C.

Table 1
Temperature on the inner and outer surface of the water-wall tube calculated by exact and numerical methods

Angle [deg]	T_{in}-exact [°C]	T_{in}-num. [°C]	Error E [%]	T_o-exact [°C]	T_o-num. [°C]	Error E [%]
0	325.22	325.21	2.07E-03	354.69	354.66	8.21E-03
15	325.07	325.06	1.84E-03	353.93	353.89	1.06E-02
30	324.58	324.58	1.68E-03	351.48	351.44	1.03E-02
45	323.72	323.72	1.74E-03	347.02	346.97	1.35E-02
60	322.51	322.51	1.59E-03	340.63	340.62	3.90E-03
75	321.15	321.16	−2.17E-03	333.38	333.38	−2.60E-04
90	319.96	319.98	−4.69E-03	327.07	327.1	−9.95E-03
105	319.21	319.22	−3.57E-03	323.14	323.19	−1.53E-02
120	318.91	318.91	−1.35E-04	321.71	321.68	8.18E-03
135	318.90	318.9	−1.37E-03	321.76	321.76	−5.26E-04
150	318.97	318.97	−6.94E-04	322.19	322.19	3.72E-04
165	319.02	319.02	3.63E-04	322.47	322.46	3.98E-03
180	319.04	319.04	−7.92E-04	322.55	322.55	7.34E-04

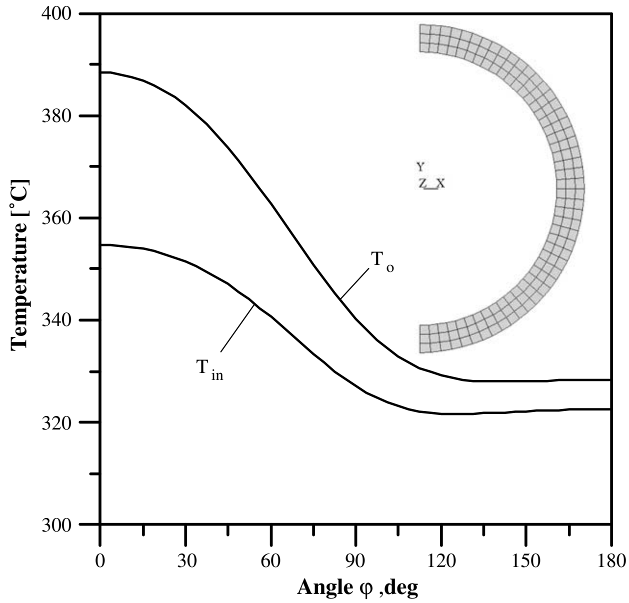

Fig. 8. Temperature distribution on the inner and outer surface of water-wall tube calculated by numerical methods.

The "measured" five temperatures were artificially generated by ANSYS software using the FEM.

The following input parameters were assumed:

- $\dot{q}_m = 300,000\ \mathrm{W/m^2}$,
- $h = 5000\ \mathrm{W/(m^2K)}$,
- $T_f = 320\ ^\circ\mathrm{C}$.

Temperature distribution on the inner and outer surface of water-wall tube is presented in Fig. 8. Fig. 9 shows a temperature distribution on the whole cross-section.

Artificially generated temperatures, necessary for identification of thermal boundary conditions in water-wall tubes, are: $f_1 = 419.75\ ^\circ\mathrm{C}$, $f_2 = 396.58\ ^\circ\mathrm{C}$,

$$f_3 = 394.79\ ^\circ\mathrm{C},\quad f_4 = 417.45\ ^\circ\mathrm{C},\quad f_5 = 332.79\ ^\circ\mathrm{C}.$$

The inverse problem is solved in an iterative way, starting form initial values:

$$\dot{q}_m^{(0)} = 250,000\ \mathrm{W/m^2},\quad h^{(0)} = 4000\ \mathrm{W/(m^2K)},\quad T_f^{(0)} = 319\ ^\circ\mathrm{C}.$$

After 10 iterations the following values were obtained:

$$\dot{q}_m^{*} = 299999.8\ \mathrm{W/m^2},\quad h^{*} = 4999.99\ \mathrm{W/(m^2K)},\quad T_f^{*} = 320\ ^\circ\mathrm{C}.$$

There is only a small difference between the estimated parameters and the input values.

The uncertainties of the obtained results are similar to those calculated for the constant thermal conductivity.

6. Conclusions

The method presented in the paper can be used for simple and accurate determining of absorbed heat flux, inner heat transfer coefficient and fluid temperature in water-walls of combustion chambers. The unknown parameters associated with the solution of the inverse heat conduction problem (IHCP) are selected to achieve the closest agreement in a least squares sense between the computed and measured temperatures using the Levenberg–Marquardt method. The temperature distribution in the water-wall tube was determined analytically for constant thermal conductivity of tube material and numerically when thermal conductivity of tube material is a function of temperature.

The uncertainties in the estimated parameters are calculated using the error propagation rule of Gauss. The method developed can be easily extended for monitoring of scale deposition on the tube inner surfaces and fire side fouling of water-walls in combustion chambers. In order to increase the accuracy of parameter determining the thickness of the measuring tube

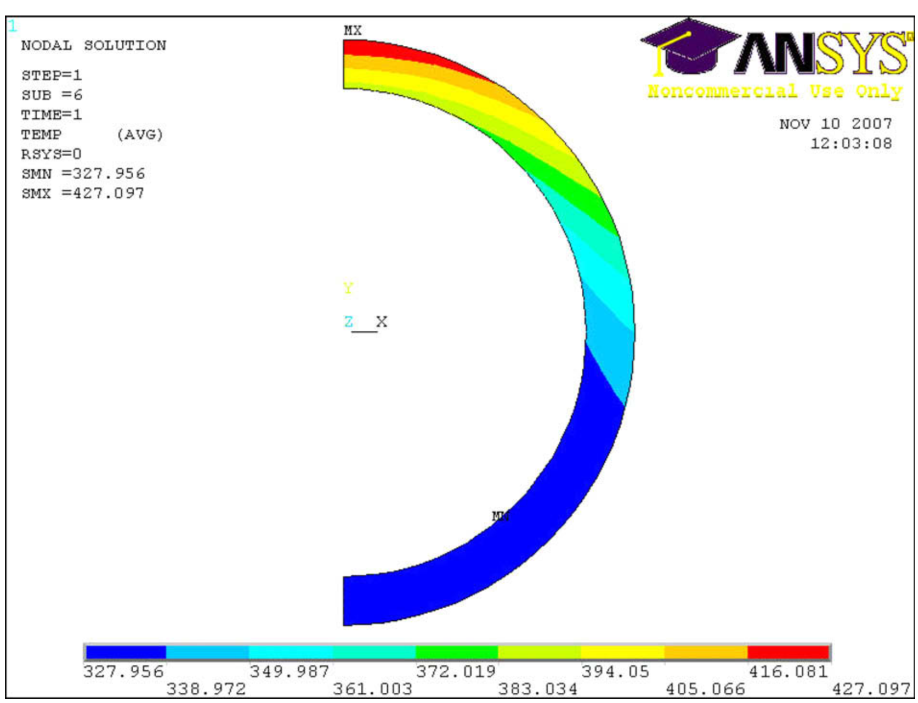

Fig. 9. Temperature distribution in °C in water-wall tube cross-section obtained by using ANSYS code.

can be enlarged to reduce an error resulting from the small distance between temperature sensors at the fireside part of the measuring tube.

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